Numerical Simulation of Fluid Motion, ed. John Noye
pp. 371-381

SENSITIVITY ANALYSIS OF A TROPICAL CYCLONE SURGE MODEL

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SUMMARY

Various numerical aspects of an explicit finite-difference model for tropical cyclone storm surge generation and propagation are investigated numerically. The open boundary conditions pose a particular problem as the geophysical scale of a cyclone normally exceeds the area that can feasibly be modelled on a digital computer. Four alternative open boundary conditions are discussed. The relationship between the geophysical scale of the cyclone and the model grid spacing is also considered. The paper concludes with a discussion of the interaction among the various terms of the governing equations.

1. INTRODUCTION

The numerical hydrodynamic model SURGE has been developed on the James Cook University DEC System-10 to study tropical cyclone storm surge generation and propagation along the Queensland coast. Numerical modelling of long wave propagation is not a new problem; satisfactory computations of astronomical tide propagation in seas or wide estuaries were first undertaken in the 1950's. There are however several unique aspects of the present problem that warranted detailed investigation. These problems stem from the complex character and geophysical extent of the meteorological forcing and include the specification of suitable open boundary conditions and the relationship between the length scale of the tropical cyclone and the model grid spacing.

2. THE NUMERICAL HYDRODYNAMIC MODEL

Long Wave Equations

Storm surge propagation is governed by the Long Wave Equations (Welander, 1964), expressing respectively the conservation of mass and the conservation of momentum in spatial directions x and y:

$$\frac{\partial \eta}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \qquad (2.1a)$$

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}\left(\frac{U^2}{h+\eta}\right) + \frac{\partial}{\partial y}\left(\frac{UV}{h+\eta}\right) - fV = -g(h+\eta)\frac{\partial \eta}{\partial x} - \frac{h+\eta}{\rho_w}\frac{\partial p_s}{\partial x} + \frac{1}{\rho_w}(\tau_{xs} - \tau_{xb}) \qquad (2.1b)$$

$$\frac{\partial V}{\partial t} + \frac{\partial}{\partial x}\left(\frac{UV}{h+\eta}\right) + \frac{\partial}{\partial y}\left(\frac{V^2}{h+\eta}\right) + fU = -g(h+\eta)\frac{\partial \eta}{\partial y} - \frac{h+\eta}{\rho_w}\frac{\partial p_s}{\partial y} + \frac{1}{\rho_w}(\tau_{ys} - \tau_{yb}) \qquad (2.1c)$$

The xy datum plane is located at the mean water level with the z axis directed vertically upwards. The water surface elevation with respect to datum is $\eta(x,y,t)$, the sea bed is $h(x,y)$ below datum, U and V are depth-integrated flow velocities or discharges per unit width, f is the Coriolis parameter, and τ_b is bed friction.

The hydrodynamic forcing influence of the tropical cyclone is introduced into the governing equations by the specification of suitable values for the wind stress τ_s and atmospheric pressure p_s at the water surface. For this purpose it is necessary to adopt a suitable model of the atmospheric boundary layer that will allow the estimation of spatial and temporal variations in the shear stress and pressure at the water surface. Suitable models have been proposed by the National Hurricane Research Project of the U.S. Weather Bureau, (Graham and Nunn (1959), Jelesnianski (1965), and U.S. Weather Bureau (1968)). The simplified model of Jelesnianski (1965) was adopted for the present purposes, although the more realistic model of the U.S. Weather Bureau (1968) will be used in production runs.

Typical radial profiles of M.S.L. pressure and sustained azimuthal (or tangential) wind speed at height 10 m are shown in Figure 1. The principal features of these profiles are the sharp gradients around the radius of maximum wind R and the much more gradual changes beyond this position; in fact the

spatial extent of the organised motion within the atmospheric boundary layer can extend some 1000 km from the eye.

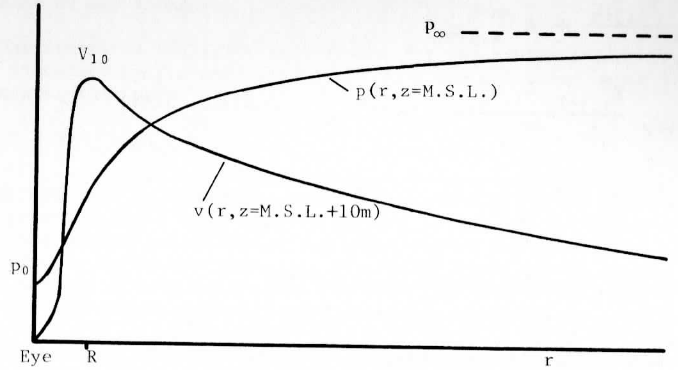


FIGURE 1. TYPICAL WIND VELOCITY AND PRESSURE PROFILES WITHIN A TROPICAL CYCLONE

Finite Difference Equations

Discrete values of the variables are specified on a space (x,y) and time (t) staggered grid, whose node points are defined as $(i\Delta x, j\Delta y, n\Delta t)$. Depth h or D below M.S.L. and water surface elevation η or H are located at (i,j,n), depth-integrated flow U at $(i+\frac{1}{2},j,n+\frac{1}{2})$ and depth-integrated flow V at $(i,j+\frac{1}{2},n+\frac{1}{2})$. The finite difference equations, which constitute an explicit leap-frog operator; are:

(a) x momentum equation centred at $(i+\frac{1}{2},j,n)$

$$\frac{U^{n+\frac12}_{i+\frac12,j} - U^{n-\frac12}_{i+\frac12,j}}{\Delta t} - f\,\bar{\bar{V}}^{n-\frac12}_{i+\frac12,j} = -g\,\overline{DX}^{n}_{i+\frac12,j}\,\frac{H^{n}_{i+1,j} - H^{n}_{i,j}}{\Delta x} - g\,\overline{DX}^{n}_{i+\frac12,j}\,\frac{B^{n}_{i+1,j} - B^{n}_{i,j}}{\Delta x} + WX^{n}_{i+\frac12,j} - \frac{\lambda}{8}\sqrt{\left(U^{n-\frac12}_{i+\frac12,j}\right)^2 + \left(\bar{\bar{V}}^{n-\frac12}_{i+\frac12,j}\right)^2}\;\frac{U^{n+\frac12}_{i+\frac12,j}}{\left(\overline{DX}^{n}_{i+\frac12,j}\right)^2} \qquad (2.2a)$$

(b) y momentum equation centred at $(i,j+\frac{1}{2},n)$

$$\frac{V^{n+\frac12}_{i,j+\frac12} - V^{n-\frac12}_{i,j+\frac12}}{\Delta t} + f\,\bar{\bar{U}}^{n-\frac12}_{i,j+\frac12}$$

$$= -g\,\overline{DY}^n_{i,j+\frac12}\,\frac{H^n_{i,j+1} - H^n_{i,j}}{\Delta y} - g\,\overline{DY}^n_{i,j+\frac12}\,\frac{B^n_{i,j+1} - B^n_{i,j}}{\Delta y}$$

$$+ WY^n_{i,j+\frac12} - \frac{\lambda}{8}\sqrt{(\bar{\bar{U}}^{n-\frac12}_{i,j+\frac12})^2 + (V^{n-\frac12}_{i,j+\frac12})^2}\;\frac{V^{n+\frac12}_{i,j+\frac12}}{(\overline{DY}^n_{i,j+\frac12})^2} \qquad (2.2b)$$

(c) continuity equation centred at $(i,j,n+\frac{1}{2})$

$$\frac{H^{n+1}_{i,j} - H^n_{i,j}}{\Delta t} + \frac{U^{n+\frac12}_{i+\frac12,j} - U^{n+\frac12}_{i-\frac12,j}}{\Delta x} + \frac{V^{n+\frac12}_{i,j+\frac12} - V^{n+\frac12}_{i,j-\frac12}}{\Delta y} = 0 \qquad (2.2c)$$

where $H^n_{i,j} = H(i\Delta x, j\Delta y, n\Delta t)$

$U^{n+\frac12}_{i+\frac12,j}$, $V^{n+\frac12}_{i,j+\frac12}$ similarly

$$B^n_{i,j} = \frac{1}{\rho_w g}\,p_s(i\Delta x, j\Delta y, n\Delta t)$$

$$WX^n_{i+\frac12,j} = \frac{1}{\rho_w}\,\tau_s\,((i+\tfrac12)\Delta x, j\Delta y, n\Delta t)$$

$WY^n_{i,j+\frac12}$ similarly

$\Delta x = \Delta y = \Delta s$ = spatial grid dimension

Δt = time step, λ = friction factor

$$\overline{DX}^n_{i+\frac12,j} = \tfrac12(D_{i,j} + H^n_{i,j} + D_{i+1,j} + H^n_{i+1,j})$$

$\overline{DY}^n_{i,j+\frac12}$ similarly

$$\bar{\bar{U}}^{n-\frac12}_{i,j+\frac12} = \tfrac14(U^{n-\frac12}_{i-\frac12,j} + U^{n-\frac12}_{i+\frac12,j} + U^{n-\frac12}_{i+\frac12,j+1} + U^{n-\frac12}_{i-\frac12,j+1})$$

$\bar{\bar{V}}^{n-\frac12}_{i+\frac12,j}$ similarly

The finite difference approximations to the convective terms are consistent with the above equations and have been omitted for simplicity. Reduced forms of these equations are necessary in the neighbourhood of open (H) and closed (U,V) boundaries, and the Great Barrier Reef.

Stability and Wave Deformation Characteristics of the Linearised Equations

Formal analytical evaluations of stability and wave deformation can only be attempted for the linearised equations. Terms such as bed friction, convective accelerations and meteorological forcing cannot be included, and it must be assumed that the computational segment considered is far away from the disturbing influence of any boundary.

The numerical stability requirement for the explicit leap-frog scheme was first discussed by Platzman (1956) who gave the following upper limit for the incremental time step:

$$\Delta t \not> \Delta s\ /\ (2gh_{max})^{\frac12} \qquad (2.3)$$

In addition to stability, numerical distortion of the physical surge wave needs investigation. Some measure of this distortion can be had from a comparison of the numerical and analytical solutions of the linearised equations. An investigation of various numerical schemes by Sobey (1970), using the propagation factor concept of Leendertse (1967) showed that the present scheme has negligible amplitude distortion when the above stability condition is satisfied.

The numerical deformation characteristics of the full long-wave equations are beyond analytical evaluation and can only be investigated by numerical experiment. A number of such experiments under various conditions have been completed. The most important objectives were to establish the applicability of the Courant condition as an estimate of the maximum incremental time step, the suitability of different open boundary conditions, the resolution capability of the grid spacing and the interaction of the various terms of the conservation equations. The former objective was adequately demonstrated and will not be further considered.

3. OPEN BOUNDARY CONDITIONS

In terms of meteorological tides, the spatial extent of a tropical cyclone would approach 1000 km, although the region of peak positive and negative surges would have a spatial scale of the order of the radius of maximum winds (typically 25 km). Ideally, the open boundaries of the computation field should be sufficiently distant from the cyclone that boundary water levels can be taken as mean sea level (or tide level). However, the storage limitations of modern time-sharing computer systems preclude the adoption of a computational field that has linear dimensions of the order of 1000 km, and simultaneously reproduces details on the scale of the radius of maximum winds. A practical compromise to this conflict of scales must be adopted and the forcing influence of the cyclone outside the computational field must be included in the open boundary conditions; the problems associated with computer storage limitations are discussed in the following section.

Four possible open boundary conditions are briefly discussed and an evaluation of their respective performances is made. Open boundary conditions can be either a water level or flow condition. The alternatives considered were:

(a) Open boundary water levels set equal to the pressure surge, the head of water equivalent to the pressure deficit, i.e. -

$$H^n_{i,j} = \frac{p_\infty - p^n_{s\,i,j}}{\rho_w g} \qquad (3.1)$$

(b) Open boundary water levels set to M.S.L. (Heaps, 1969), i.e. -

$$H^n_{i,j} = 0 \qquad (3.2)$$

(c) Flow gradient perpendicular to open boundary set to zero (Jelesnianski, 1965), e.g. -

$$V^{n+\frac{1}{2}}_{i,j+\frac{1}{2}} = V^{n+\frac{1}{2}}_{i,j-\frac{1}{2}} \qquad (3.3)$$

at the northern open boundary of a grid.

(d) Open boundary water levels set equal to the local bathystrophic storm tide, i.e., the quasi-static profile described, for example by -

$$0 = -g(h+\eta)\frac{\partial \eta}{\partial x} - \frac{h+\eta}{\rho_w}\frac{\partial p_s}{\partial x} + \frac{\tau_{xs}}{\rho_w} \qquad (3.4)$$

when the open boundary is in the x direction.

For comparison purposes, a hypothetical continental shelf region extending 195 n mile (361 km) along a length of straight coast was considered. The sea bed sloped uniformly at a slope of 0.00055 away from a depth of 2.0 m at the coast. At the outer open boundary, 137.5 n mile (255 km) from the coast, the depth was 145 m. The shelf region was assumed centred about 18°S latitude. A test cyclone, similar in many respects to tropical cyclone Althea (Townsville, 1971), was adopted. The cyclone was initially placed in the centre of the flow field and the model allowed a period of 10 iterations for initial build-up from still water before the cyclone was moved perpendicular to the coast at the nominated speed. Total simulation time was 5 hours, with landfall after 3¼ hours. Grid spacing was 5 n mile and the time step was 150 secs.

The first of the four boundary conditions was selected as a basis for comparison. Using this condition, a peak surge of 3.3 m occurred approximately 30 minutes after cyclone landfall and 20 km south of the landfall position. Encouragingly, the phase and amplitude of the peak surge in all cases was almost identical. Such was not the case however away from the peak surge location and close to the open boundaries. Ideally, realistic simulation as close as possible to the open boundaries is required. Detailed comparisons of the relative performance of the boundary conditions can be found in Sobey et al (1975). Figure 2 reproduces some typical coastal water level profiles (observer facing seawards) and the overall behaviour is summarised below.

Mean Sea Level Condition (Figure 2a)

The numerical simulations with boundary conditions (a) and (b) gave broadly equivalent flow patterns; phase agreement was generally good but water level differences averaged 20 cm with a maximum of some 35 cm over the field (Figure 2a). It appeared that the times immediately before and after landfall of the cyclone were most sensitive to these water level deviations, with recovery taking place approximately an hour later. This level decrease could in part be due to a widespread drawdown around the eye for the M.S.L. boundary condition.

Asymptotic Flow Condition (Figure 2b)

Although the peak amplitude and phase matching was good throughout, significant water level differences occurred near the open boundaries. Figure 2b is a typical result, drawdown north of the landfall position and build-up to the south both being accentuated.

The original appeal of this condition (c) was its ability to allow variations in water level at the open boundaries, hopefully producing a more realistic flow pattern in these regions. However, spurious effects were observed in the

flow patterns, particularly at intersections of open boundaries. Physically, it is difficult to see what is particularly significant about the flow gradient normal to an open boundary, as the position and especially the orientation of this boundary are essentially arbitrary.

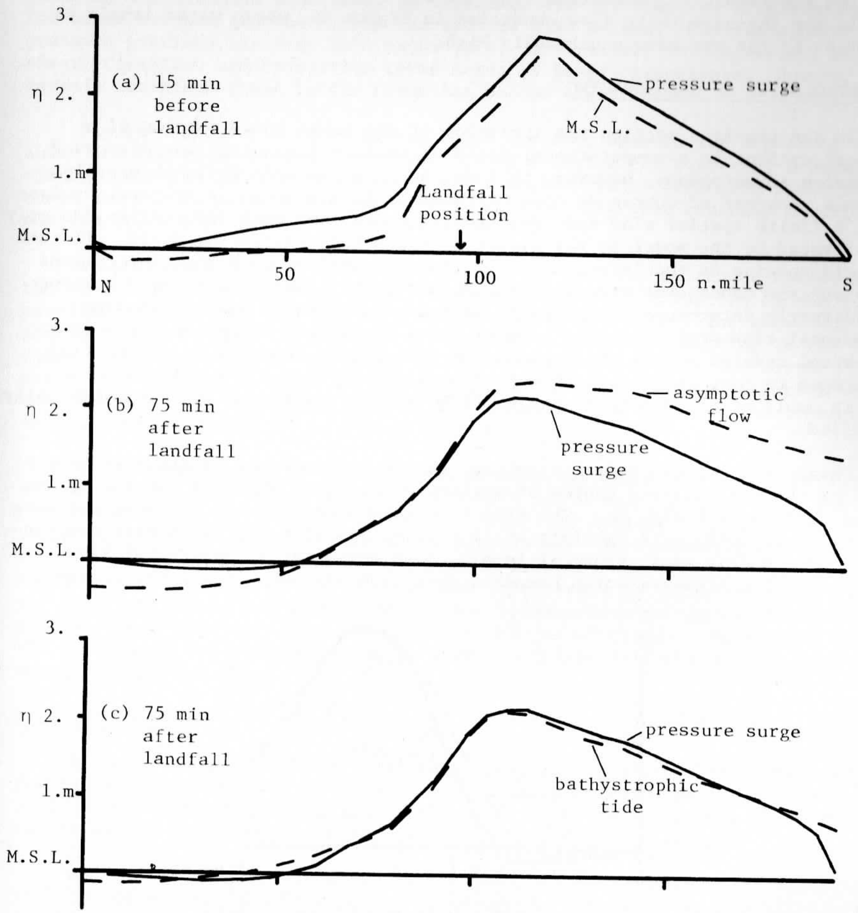


FIGURE 2. COMPARATIVE PERFORMANCE OF OPEN BOUNDARY CONDITIONS

Bathystrophic Storm Tide Approximation (Figure 2c)

Again from a physical viewpoint the bathystrophic tide condition (d) should provide the most realistic boundary condition as it involves a lowest order momentum balance along the open boundary; in practice it has proved very effective. The bathystrophic tide profile would only be used along those open boundaries which encounter land at some point and it would be used in conjunction with the pressure surge condition (a) in deep water.

Basic flow patterns and peak surge phase and amplitude remained unchanged throughout the simulation and resulting water surface contour patterns were well behaved and appeared the most plausible of the four results. Figure 2c, showing the resultant surge profile along the coast 75 minutes after cyclone landfall typifies the result for condition (d). It can be comapred to the corresponding profile for the asymptotic flow condition in Figure 2b, where water levels to the right of the eye seem unnaturally high.

4. RESOLUTION OF GRID SPACING

In the previous section the inclusion of the large scale forcing of a tropical cyclone in a computational field of smaller dimensions was discussed. It remains advantageous, however, to model as large an area as is physically possible, subject of course to computer storage limitations. A tropical cyclone has a definite spatial wind and pressure structure, and this information can only be delivered to the model at the discrete grid points within the field. If then the grid spacing is too large, possible loss in resolution of the true cyclone behaviour and consequent wave deformation may result. The adopted grid spacing also directly determines the computer storage requirements and the maximum incremental time step (explicit scheme) for a particular situation. The large time step, coupled with a small number of grid points, results in substantial advantages in computation time for a 10 n mile grid over a 5 n mile grid, so that it would be preferable to use as large a grid spacing as is hydrodynamically justified.

These problems are directly related to the ratio of the fundamental length scale of the cyclone, the radius of maximum winds R, to the grid spacing of the finite difference field, Δs. The effect of this $R/\Delta s$ ratio on the computed wave was evaluated using grid spacings of 10 n mile, 5 n mile and 3.3 n mile respectively; the models were identical in all other respects, in particular R was 19 n mile. The pressure surge boundary condition was used in all three cases.

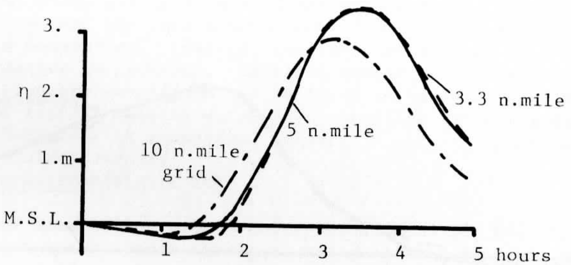


FIGURE 3. TYPICAL GRID RESOLUTION RESULT

Figure 3 is a typical result, and shows the water level histories at the position of the peak surge. It appears that decreasing the $R/\Delta s$ ratio results in a lowering of the peak surge level, with an associated phase advance of the computed wave. Because of the large variations in peak levels and phase away from the other two cases, the 10 n mile grid was considered unsatisfactory for the prediction of the peak surge at the coast. However the results show that it is capable of adequately modelling the pressure surge in deep water, which may be useful in certain cases. Additionally there appears little advantage in using the smaller 3.3 n mile grid in place of the 5 n mile grid, especially when interest centres on the phase and amplitude of the peak surge. However, if finer resolution of coastal detail is required, then a smaller grid must be used.

5. INTERACTION OF TERMS

A useful and instructive part of a comprehensive sensitivity analysis is the observation of the relative importance of the terms of the conservation equations at particular locations within the computational field. The largest terms in the momentum equations, as expected, are the temporal accelerations, the water surface gradients, and the surface wind stress. The atmospheric pressure gradient can also be a major term, particularly in deep water, with bed shear, Coriolis, and convective terms being of lesser importance. However, at certain locations these latter terms can become significant.

Figure 4 shows the interactions which occurred between the dependent variables and the terms of the y momentum equation during the course of a normal simulation. The location considered is in shallow water, one grid point off the coast and in the path of the eye of the cyclone. The open boundary condition was condition (d), the local bathystrophic tide.

At this particular location the passage of the cyclone can be observed in the behaviour of the forcing terms, in particular the fluctuation in the surface pressure gradient and the reversal of the wind stress term. The water surface gradient term is seen to steadily increase in magnitude, resulting in the characteristic set-up to the south of the eye and set-down to the north. Of particular interest here is the increasing influence of the bed friction term in this shallow water location, and the importance of the Coriolis and convective accelerations especially as the eye passes.

An examination of the interactions between the dependent variables shows that in the initial stages flow was basically north-east and away from the coast, driven by the strong circular wind field about the cyclone. This resulted in a slight drop of the water surface below mean level. Approximately one hour before the eye passed, however, the x component of flow (U) was directed towards the coast and the surge wave (η) build-up was initiated. An important point is that the peak surge lags the passing of the eye by some twenty minutes, followed by a strong return flow away from the coast in the latter stages. Mechanical energy (E) throughout the simulation can be seen to closely follow the behaviour of the water surface elevation.

6. CONCLUSIONS

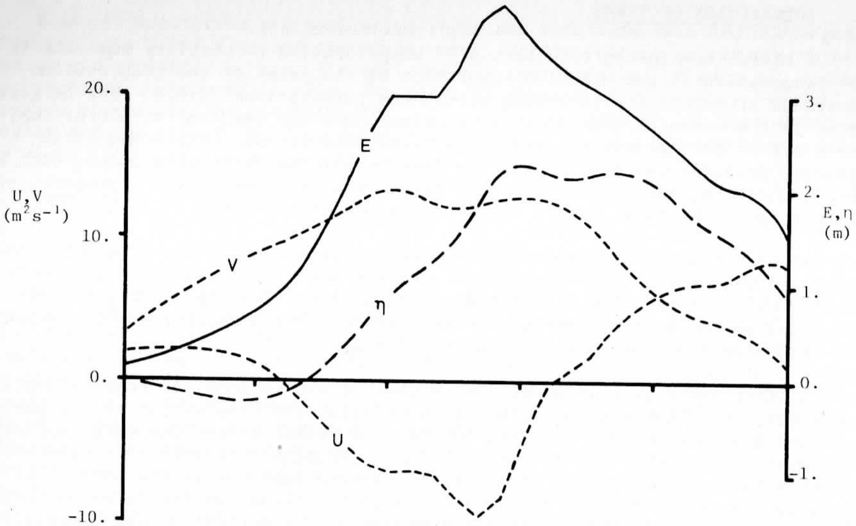
A numerical hydrodynamic model that adequately describes the generation and propagation of tropical cyclone storm surge has been developed. Variations in cyclone intensity (deepening and filling) and in speed and direction of the eye can be varied continuously (after each time step if necessary).

A detailed sensitivity analysis has resolved the open boundary condition problem, demonstrating the suitability of the bathystrophic tide condition in representing the forcing influence of the tropical cyclone beyond the boundaries of the computational field. Realistic flow patterns can now be obtained in the vicinity of the open boundaries.

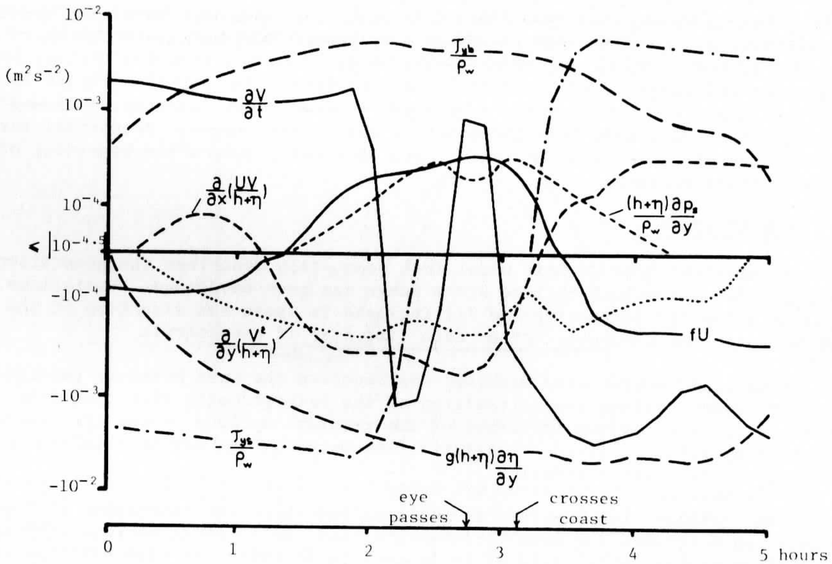
Additionally, the sensitivity analysis has shown the importance of the scale relationship between the model grid spacing and the tropical cyclone dimensions. It appears that an $R/\Delta s$ ratio of at least 4 is necessary to adequately represent the meteorological forcing.

7. ACKNOWLEDGEMENTS

This study was conducted as part of a sponsored research project on tropical cyclone storm surge generation and propagation along the Queensland coast. Financial support from the Beach Protection Authority, Brisbane, and the



(a) Dependent Variables and Mechanical Energy



(b) Terms of y-Momentum Equation

FIGURE 4. SHALLOW WATER MOMENTUM BALANCE DURING PASSAGE OF CYCLONE

co-operation of the James Cook University Computer Centre are gratefully acknowledged.

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